Causal Classification of Spatiotemporal Quantum Correlations

Minjeong Song,[1,*] Varun Narasimhachar,[1,2,†] Bartosz Regula,[3] Thomas J. Elliott,[4,5] and Mile Gu[1,6,7,‡]

[1]*Nanyang Quantum Hub, School of Physical and Mathematical Sciences, Nanyang Technological University, 637371, Singapore*
[2]*Institute of High Performance Computing, Agency for Science, Technology and Research (A*STAR), 1 Fusionopolis Way, Singapore 138632*
[3]*Mathematical Quantum Information RIKEN Hakubi Research Team, RIKEN Cluster for Pioneering Research (CPR) and RIKEN Center for Quantum Computing (RQC), Wako, Saitama 351-0198, Japan*
[4]*Department of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
[5]*Department of Mathematics, University of Manchester, Manchester M13 9PL, United Kingdom*
[6]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore*
[7]*MajuLab, CNRS-UNS-NUS-NTU International Joint Research Unit, UMI 3654, Singapore 117543, Singapore*

(Received 8 July 2023; accepted 12 August 2024; published 11 September 2024)

From correlations in measurement outcomes alone, can two otherwise isolated parties establish whether such correlations are atemporal? That is, can they rule out that they have been given the same system at two different times? Classical statistics says no, yet quantum theory disagrees. Here, we introduce the necessary and sufficient conditions by which such quantum correlations can be identified as atemporal. We demonstrate the asymmetry of atemporality under time reversal and reveal it to be a measure of spatial quantum correlation distinct from entanglement. Our results indicate that certain quantum correlations possess an intrinsic arrow of time and enable classification of general quantum correlations across space-time based on their (in)compatibility with various underlying causal structures.

DOI: 10.1103/PhysRevLett.133.110202

Consider Alice and Bob, situated in their own laboratories. In each round, they each receive correlated random variables A and B. The correlations were distributed via one of three possible causal mechanisms: (i) spatially such that A and B share a common cause, (ii) temporally such that measurement outcomes of A are communicated to B or vice versa, and (iii) some combination of the above (see Fig. 1). Alice and Bob record these correlations. With only this recording, can we rule out one of the causal mechanisms above? Classical statistics says no. The observed correlations will always be compatible with all possible scenarios. Thus the adage "correlation does not imply causation."

Quantum correlations can exhibit remarkable differences. Suppose Alice and Bob each receive a single qubit each round, which they then measure in some Pauli basis. The correlations between their measurement outcomes can lie outside what a density operator describes. Such "aspatial correlations" cannot be explained purely by a common cause [see Fig. 1(a)], leading to quantum correlations that can indeed imply causation [4,5]. There is thus significant interest in quantum causal inference [4–11] due to its stark departure from classical statistics.

Here we ask, do certain quantum correlations require a common cause? We answer in the affirmative by formalizing the notion of "atemporal" quantum correlations—correlations between Alice and Bob's qubit measurements that cannot be explained by purely temporal means [i.e., as two measurements on a qubit communicated by some quantum channel between Alice and Bob; see Fig. 1(b)]. We demonstrate computable necessary and sufficient indicators for atemporality and demonstrate that (1) it is

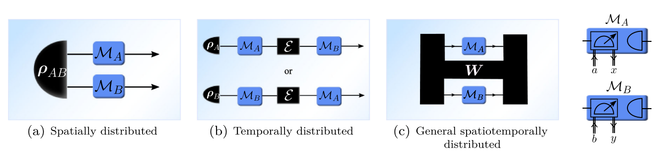

FIG. 1. Causal distribution mechanisms. Consider Alice and Bob that make respective projective measurements $\mathcal{M}_A$ and $\mathcal{M}_B$ on respective quantum systems A and B (blue boxes). We divide potential mechanisms of correlating these measurements into three categories: (a) purely spatially distributed mechanisms involving A and B being two aims of some bipartite state ρ_{AB}, reflecting the case of common cause; (b) purely temporally distributed such that A and B represent the input and output of some quantum channel $\mathcal{E}$, reflecting the case of direct cause; or (c) some combination of both. Examples include non-Markovian evolution [1,2] or cases of indefinite causal order [3]. We refer to correlations that are incompatible with (a) as aspatial and those incompatible with (b) as atemporal.

*Contact author: song.at.qit@gmail.com
†Contact author: varun.achar@gmail.com
‡Contact author: mgu@quantumcomplexity.org

asymmetric under time reversal and thus reveals the existence of correlations possessing an intrinsic arrow of time, and (2) it represents a new operational form of nonclassical correlations distinct from entanglement. This induces a framework for "causal classification"—classifying general spatiotemporal quantum correlations based on their compatibility with various causal mechanisms. Our results thus provide new mathematical tools and concepts for understanding how quantum correlations can infer causal structure in ways without classical analogs.

Framework—Returning to Alice and Bob in their own laboratories, we label the qubits Alice and Bob each possess respectively by A and B. We assume that the qubit pair is prepared in the same way in each round. Such a preparation scheme may be (i) "spatially distributed" such that A and B correspond to two parts of some bipartite state $\boldsymbol{\rho}_{AB}$ [see Fig. 1(a)], (ii) "temporally distributed" such that B is the output of A subject to some fixed quantum channel (completely positive trace-preserving map) $\mathcal{E}$ or vice versa [see Fig. 1(b)], or (iii) neither purely spatially nor temporally distributed such A and B are related by general "process matrices" [see Fig. 1(c)]. A special case being "non-Markovianity," where evolution from A to B involves coupling from an ancillary environment E that is initially correlated with A [1,2,12].

Let $\boldsymbol{\sigma}_0 = \mathbf{I}, \boldsymbol{\sigma}_1 = \mathbf{X}, \boldsymbol{\sigma}_2 = \mathbf{Y}, \boldsymbol{\sigma}_3 = \mathbf{Z}$ be the identity and the three standard Pauli operators [13]. Let $\Pr(x, y|a, b)$ then denote the probability of Alice getting outcome x and Bob getting outcome y when Alice chooses to measure in basis $\boldsymbol{\sigma}_a$ and Bob in $\boldsymbol{\sigma}_b$. Alice and Bob do not perform any other interventions. By choosing appropriate Pauli measurements over a large number of rounds, Alice and Bob can determine the expectation values $\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle = \sum_{x,y} xy \Pr(x, y|a, b)$ describing how their measurement outcomes correlate in various Pauli basis to any desired level of accuracy. Alice and Bob then pass this information to us. What can we conclude about the causal distribution mechanisms behind the preparation of A and B?

Given Pauli correlations $\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle$ for each $a, b \in \{0, 1, 2, 3\}$, Ref. [5] proposed a concise description of this information via the pseudodensity operator (PDO)

$$\mathbf{R}_{AB} \equiv \sum_{a,b=0}^{3} \frac{\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle}{4} \boldsymbol{\sigma}_a \otimes \boldsymbol{\sigma}_b. \quad (1)$$

Initially proposed to identify quantum correlations that imply causality, it has seen significant uses toward building quantum information theories that place space and time on equal footing [9,14–23]. They contain all the information about $\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle$, since the latter can be retrieved directly via $\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle = \mathrm{tr}[\mathbf{R}_{AB}(\boldsymbol{\sigma}_a \otimes \boldsymbol{\sigma}_b)]$. Thus, our capacity to infer causal distribution mechanisms from the Pauli correlations coincides with our capacity to infer causal distribution mechanisms from the corresponding PDO. Also note that when qubits A and B are spatially distributed, $\mathbf{R}_{AB}$ reduces to a standard density operator. Meanwhile, their marginal distributions are always positive and describe local measurement statistics for Alice and Bob.

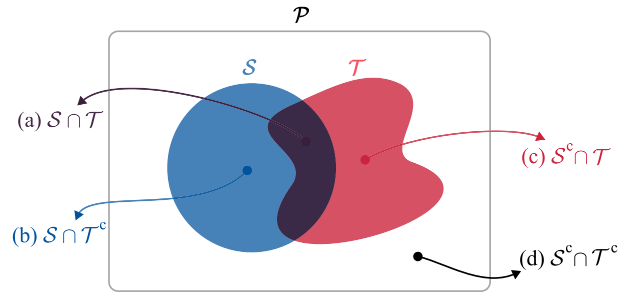

FIG. 2. Venn diagram of all spatiotemporal compatibility. The set $\mathcal{P}$ of observed spatiotemporal quantum correlations (as described by PDOs) is divided into four mutually exclusive subsets: (a) $\mathcal{S} \cap \mathcal{T}$ represents correlations that are compatible with purely spatial and purely temporal distribution mechanisms, (b) $\mathcal{S} \cap \mathcal{T}^c$ represents correlations that rule out purely temporal distribution mechanisms, (c) $\mathcal{S}^c \cap \mathcal{T}$ represents correlations that rule out purely spatial distribution mechanisms such as two coexisting qubits measured separately, and (d) $\mathcal{S}^c \cap \mathcal{T}^c$ designates correlations that require a combination of spatial and temporal distribution mechanisms to explain. Note that unlike $\mathcal{S}$, $\mathcal{T}$ does not form a convex set (see Example 5 in the Supplemental Material [24]).

Spatial and temporal compatibility—We introduce two distinct criteria on PDOs. We say that $\mathbf{R}_{AB}$ is "spatially compatible," or belongs to $\mathcal{S}$ if its statistics can be generated via a spatial distribution mechanism [i.e., as in Fig. 1(a)]. Similarly, we say that $\mathbf{R}_{AB}$ is "temporally compatible," or belongs to $\mathcal{T}$ if its statistics can be generated via a temporal distribution mechanism [i.e., as in Fig. 1(b)]. We will often use the terms "spatial" and "temporal" for brevity, but we stress that they only mean *compatible with* a spatially or temporally distributed structure. PDOs that lie outside of $\mathcal{S}$ are referred to as "aspatial," and those that lie outside of $\mathcal{T}$ are referred to as "atemporal."

We then divide the set of all PDOs using a Venn diagram into four separate classes based on their spatiotemporal compatibility: those that (a) lie in $\mathcal{S}$ and $\mathcal{T}$ and are thus compatible with any distribution mechanism, (b) lie in $\mathcal{S}$ but not $\mathcal{T}$ and thus rule out purely temporal distribution mechanisms, (c) lie in $\mathcal{T}$ but not $\mathcal{S}$ and thus rule out purely spatial distribution mechanisms, and (d) lie outside $\mathcal{S}$ and $\mathcal{T}$ and cannot be explained by either purely spatial or temporal distribution schemes but rather rely on a more complicated combination of spatial and temporal mechanisms. States that lie in (a) behave like classical probability distributions, and we cannot infer anything conclusive about their underlying causal distribution mechanism. Quantum correlations, however, permit PDOs in each of (b), (c), and (d), where certain causal distribution mechanisms can be ruled out (see Fig. 2).

To better understand what PDOs lie within each class, we need a necessary and sufficient criterion for aspatiality and atemporality. PDOs were initially introduced to study the former, with Ref. [5] showing that the negativity of $\mathbf{R}_{AB}$ is necessary and sufficient for aspatiality. Some works also looked into temporal correlations in some limited scenarios under additional assumptions, e.g., maximally mixed or full ranked initial state [4,6,9,17], or unitary evolution [7,8]. We will derive conditions without such assumptions for when a PDO is atemporal, and thus build a full picture of spatiotemporal compatibility.

Certifying atemporality—Given $\mathbf{R}_{AB}$, our goal is to determine identifiers of atemporality that rule out compatibility with temporal distribution mechanisms. Consider first a "forward atemporality" measure $\vec{f}$ that is zero if and only if $\mathbf{R}_{AB}$ has statistics consistent with a temporal distribution mechanism from A to B, and a "reverse atemporality" measure $\overleftarrow{f}$ that is zero if and only if $\mathbf{R}_{AB}$ has statistics consistent with a temporal distribution mechanism from B to A. Together they naturally induce a general "atemporality" measure $f = \min(\vec{f}, \overleftarrow{f})$ that is zero if and only if $\mathbf{R}_{AB}$ lies in $\mathcal{T}$.

We then introduce "pseudochannels," a temporal analog of pseudodensity operators. Recall that the Choi-Jamiołkowski isomorphism enables us to represent each qubit channel Λ by a Choi operator [26]

$$\chi_\Lambda \equiv (\mathcal{I} \otimes \Lambda)|\phi^+\rangle\langle\phi^+| \tag{2}$$

describing the output state when Λ is applied to one arm of a Bell state $|\phi^+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$. Here, $\mathcal{I}$ denotes the identity channel. We observe that this output does not need to be a valid spatial quantum state if Λ is not a valid quantum channel. More generally, let χ_Λ be a PDO with nonzero negativity $\mathcal{N}(\chi_\Lambda)$ (absolute sum of its negative eigenvalues). In such scenarios, Λ remains trace-preserving, Hermiticity-preserving, and linear but is no longer completely positive.

This motivates us to define "pseudochannels": linear maps that preserve trace and Hermiticity but which can be non-completely positive. Λ is then a pseudochannel, and $\mathcal{N}(\chi_\Lambda)$ provides a necessary and sufficient indicator of its "nonphysicality." Such pseudochannels provide a natural means to define $\vec{f}$ (and thus $\overleftarrow{f}$ and f). Given $\mathbf{R}_{AB}$, we first assert that it describes correlations resulting from some quantum channel $\vec{\Lambda}$ with input system A and output system B. References [9,27] demonstrated that for temporal PDOs, the associated quantum channel satisfies

$$\mathbf{R}_{AB} = \left(\mathcal{I}_A \otimes \vec{\Lambda}\right)\mathbf{K}_{AB}, \tag{3}$$

where $\mathbf{K}_{AB} \equiv \{\rho_A \otimes (\mathbf{I}_B/2), \mathbf{S}_{AB}\}$ with $\rho_A \equiv \mathrm{tr}_B \mathbf{R}_{AB}$ being the first marginal, $\{\cdot,\cdot\}$ the anticommutator, $\mathbf{I}$ the identity operator, and $\mathbf{S}$ the swap operator. In more general cases where $\mathbf{R}_{AB}$ is not necessarily a temporal PDO, we rationalize the following: when $\mathbf{R}_{AB}$ is incompatible with a temporal distribution mechanism from A to B, no such valid quantum channels exist. However, we can drop the complete positivity requirements on $\vec{\Lambda}$. In the Supplemental Material [24] (see Lemma 1), we prove that any PDO will have at least one compatible forward pseudochannel. This allows us to interpret *any* spatiotemporal correlations as resulting from a pseudochannel $\vec{\Lambda}$ acting on A to generate B (see Fig. 3). The minimal nonphysicality of such a channel then motivates our definition for "forward atemporality",

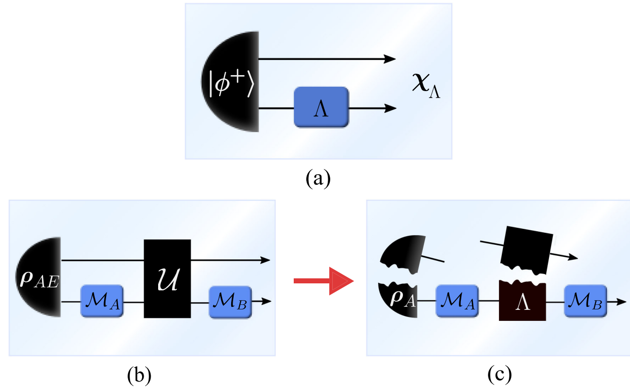

FIG. 3. Choi operator and pseudochannels. (a) The Choi operator of a quantum channel Λ describes the resulting state when we apply Λ on one arm of the Bell state $|\phi^+\rangle \equiv (1/\sqrt{2})(|00\rangle + |11\rangle)$. (b) In cases where the resulting correlations $\mathbf{R}_{AB}$ between A and B are atemporal, there is no valid quantum channel from A to B. (c) Nevertheless, we can identify a pseudochannel Λ that forcibly interprets the dynamics as a map from A to B. The resulting Λ is nonphysical, which is reflected by the negativity of its Choi operator. We show that this negativity is a necessary and sufficient condition for the forward atemporality of $\mathbf{R}_{AB}$.

$$\vec{f}(\mathbf{R}_{AB}) \equiv \min_{\vec{\Lambda}} \mathcal{N}(\chi_{\vec{\Lambda}}), \tag{4}$$

where the minimization is over all forward pseudochannels $\vec{\Lambda}$ that are compatible with $\mathbf{R}_{AB}$. Similarly, we define the "reverse atemporality" $\overleftarrow{f}$ by interchanging A and B, thus also defining the overall atemporality $f = \min(\vec{f}, \overleftarrow{f})$. For example, the entangled Bell state $(1/\sqrt{2})(|01\rangle - |10\rangle)$ has forward, reverse, and overall atemporality of 0.5 (its corresponding pseudochannel being the nonphysical universal-NOT gate [28]). This then leads to one of our key results.

Result 1—Given spatiotemporal correlations described by a PDO $\mathbf{R}_{AB}$, let

$$\vec{f}(\mathbf{R}_{AB}) \equiv \min_{\vec{\Lambda}} \mathcal{N}(\chi_{\vec{\Lambda}}), \tag{5}$$

Algorithm 1. Choi operator of pseudochannel construction.

Require: 2-qubit PDO $\mathbf{R}_{AB}$

$\boldsymbol{\rho}_A \leftarrow \mathrm{tr}_B \mathbf{R}_{AB}$

$\mathbf{L} \leftarrow [\boldsymbol{\rho}_A - (\mathbf{I}/2)] \otimes \mathrm{tr}_A\left[\left(\frac{1}{2}\boldsymbol{\rho}_A^{-1} \otimes \mathbf{I}\right)\mathbf{R}_{AB}\right] + (\mathbf{I}/2) \otimes \mathrm{tr}_A\left[\left((\mathbf{I} - \frac{1}{2}\boldsymbol{\rho}_A^{-1}) \otimes \mathbf{I}\right)\mathbf{R}_{AB}\right]$

$\boldsymbol{\chi} \leftarrow (T \otimes \mathcal{I})(\mathbf{R}_{AB} - \mathbf{L})$ ▷ T denotes the transpose map

return $\boldsymbol{\chi}$

where the minimization is over all forward pseudochannels $\vec{\Lambda}$ that are compatible with $\mathbf{R}_{AB}$ [i.e., those that satisfy Eq. (3)]. Then $f > 0$ is a necessary and sufficient condition for atemporality. Moreover, we have a systematic algorithm (see below) to compute $\vec{f}$ for any $\mathbf{R}_{AB}$.

Observe first $f > 0$ implies that no physical channel is compatible with $\mathbf{R}_{AB}$ by definition, while existence of a compatible $\vec{\Lambda}$ (as expressed by their Choi operator) can be systematically identified as follows: when $\mathbf{R}_{AB}$ has full rank marginals, $\vec{\Lambda}$ is unique and the algorithm that returns its Choi operator $\boldsymbol{\chi}$ is particularly simple (see Algorithm 1). From this, the forward atemporality can be directly computed.

When the first marginal $\boldsymbol{\rho}_A$ is rank-deficient (i.e., some pure state $|\phi\rangle\langle\phi|$), the pseudochannels compatible with $\mathbf{R}_{AB}$ are no longer unique. This is because any such causal interpretation corresponds to Alice being given a system in $|\phi\rangle$, such that measured statistics do not contain any information regarding outputs when Λ acts on a state $|\phi^\perp\rangle$ perpendicular to $|\phi\rangle$. In the Supplemental Material [24], we generalize the above algorithm to identify all compatible pseudochannels, and a semidefinite program to find the minimum nonphysicality among them. Thus, we can systematically evaluate the forward atemporality $\vec{f}$ for all two-qubit PDOs. Interchanging A and B enables evaluation of the reverse atemporality $\overleftarrow{f}$ and thus overall atemporality f.

Properties of atemporality—The computability of atemporality offers efficient means to study its properties. Here, we survey key results (see Supplemental Material [24] for further details). The first is time-reversal asymmetry. Unlike classical correlations, certain quantum correlations admit temporal mechanisms in only one temporal direction.

Result 2—Forward atemporality does not imply reverse atemporality or vice versa.

Consider a PDO describing a single qubit A undergoing probabilistic dephasing $\mathcal{E}(\boldsymbol{\rho}) \coloneqq p\boldsymbol{\rho} + (1-p)\mathbf{Z}\boldsymbol{\rho}\mathbf{Z}^\dagger$, the output of which we label qubit B. Clearly, its forward atemporality is 0 by construction. However, $\overleftarrow{f}(\mathbf{R}_{AB}) > 0$ for all $p \neq 0$, 1 (see Example 8 in the Supplemental Material [24]). Thus, quantum correlations not only can imply causality as previously suggested [4,5], but can also only imply causality in a particular temporal direction.

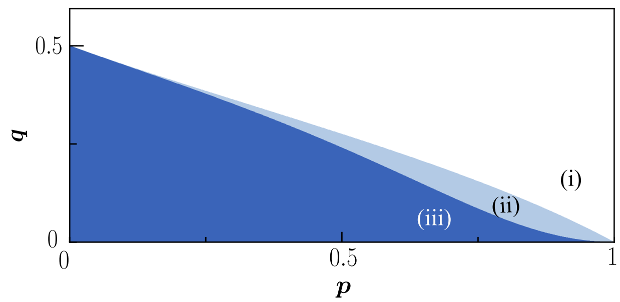

FIG. 4. Entanglement and atemporality. The family of biased Werner states $\boldsymbol{\rho}^{\mathrm{BW}}_{p,q} \equiv (1-p)\boldsymbol{\rho}^{\mathrm{Werner}}_q + p|00\rangle\langle 00|$ illustrates the differences between entanglement and atemporality. The plot depicts a color map depicting entanglement and atemporality of $\boldsymbol{\rho}^{\mathrm{BW}}_{p,q}$ for various values of p and q. While (i) all separable states are not atemporal, (ii) there exist entangled states that nevertheless admit temporal distribution mechanisms. Notwithstanding, (iii) most entangled states within the family are atemporal.

The interplay of temporal and spatial compatibility is also a natural point of interest. Specifically, let us restrict ourselves to density operators (i.e., correlations that lie in $\mathcal{S}$) and let $\{|e_j\rangle\langle e_j|\}$ and $\{|f_j\rangle\langle f_j|\}$ be some orthonormal basis respectively on A and B. In the Supplemental Material [24], we show that classical distributions of such orthogonal states, i.e., of form $\sum_{jk} p_{jk}|e_j\rangle\langle e_j| \otimes |f_k\rangle\langle f_k|$ have zero atemporality in either direction—aligning with the intuition that classical statistics cannot rule out any causal distribution mechanism without active intervention. Meanwhile states of the form $\rho_{AB} = \sum_j p_j|e_j\rangle\langle e_j| \otimes \boldsymbol{\tau}_j$, where $\boldsymbol{\tau}_j$ is an arbitrary state on B (i.e., those with zero one-way discord [29–31]) has zero forward atemporality.

One might also speculate that entanglement implies atemporality. Indeed, we show whenever $\mathbf{R} \in \mathcal{S}$ is pure or has maximally mixed marginals (see Theorem 3 in the Supplemental Material [24]), nonzero atemporality coincides with nonzero entanglement. However, this does not hold in more general conditions.

Result 3—Entanglement does not imply atemporality: certain entangled states are temporally compatible.

Consider the parametrized family of biased Werner states $\boldsymbol{\rho}^{\mathrm{BW}}_{p,q} \equiv (1-p)\boldsymbol{\rho}^{\mathrm{Werner}}_q + p|00\rangle\langle 00|$, achieved by mixing a standard Werner state $\boldsymbol{\rho}^{\mathrm{Werner}}_q$ [32] with the state $|00\rangle\langle 00|$ (see Fig. 4). Here, $\boldsymbol{\rho}^{\mathrm{BW}}_{0.5,0.25}$ has zero atemporality, but nonzero entanglement negativity ($\approx$0.0087) [33]. Nevertheless, sufficiently strong entanglement does guarantee atemporality. In the Supplemental Material [24] (see Theorem 4 within), we prove the following.

Result 4—Any temporally compatible two-qubit state must have entanglement negativity of at most $\frac{1}{2}(\sqrt{2}-1)$.

Indeed a scatter plot of atemporality vs entanglement negativity for 1000 randomly generated density operators suggests that two concepts are heavily correlated but not the same—with atemporality looking to be a stronger notion of nonclassical correlations than entanglement (see the Supplemental Material [24]). Thus, we anticipate that future study of atemporality could well lead to a new and finer-grained understanding of quantum correlations.

Discussion—Spatiotemporal quantum correlations differ crucially from classical counterparts in that they can be fundamentally incompatible with certain underlying causal distribution mechanisms. In this Letter, we showed that such correlations between various Pauli measurements on two qubits A and B can be *atemporal*, such that their explanation necessitates some common cause. We provided a necessary and sufficient indicator of atemporality and a systematic algorithm to compute it. In studying atemporality, we illustrated (1) the existence of temporal asymmetry whereby certain correlations admit purely causal explanations in only one temporal direction and (2) that *atemporality* induces a notion of quantum correlations distinct from entanglement. Combined with prior work showing quantum correlations can also be *aspatial* such that they cannot be purely explained by a common cause, our results enable a framework to classify quantum correlations based on their compatibility with *spatial* and *temporal* mechanisms.

This classification opens a number of interesting directions. Our work here focused on the spatiotemporal correlation between two qubits as it allowed for closed-form expressions for atemporality; however, the fundamental concept introduced applies to arbitrary bipartite systems. Fundamentally, we can define the atemporality of any correlations between A and B (or vice versa) as how nonphysical a quantum channel from A and B (or vice versa) must be to generate the correlations observed. Indeed, PDOs are well-defined for bipartite systems with n-qubit partitions [21], while variants extend these ideas to general d-dimensional or continuous variable systems [34,35]. The identification of an analog of our Algorithm 1 for finding (pseudo)channels then provides a natural pathway for understanding spatiotemporal compatibility on systems of arbitrary dimensions. Meanwhile, our quantifier of atemporality used one particular measure of nonphysicality. This choice is not unique; other definitions of nonphysical maps and quantifiers of nonphysicality exist [15,36–48], and thus we can anticipate many alternative measures of atemporality akin to what exists for entanglement.

The relation between atemporality and spatial quantum correlations also yields fascinating insights. We proved that some forms of quantum correlation are required for a standard bipartite quantum system to be temporal, and sufficient entanglement guarantees atemporality. Still, many open questions remain. Some entangled states are not atemporal, so is atemporality a strictly stronger notion of quantum correlations? And if so, is it guaranteed by steering or Bell nonlocality [49]? We also have no proof that atemporality guarantees entanglement, and thus could atemporality persist in more robust forms of quantum correlations [31]? Whatever the case, atemporality has a clear operational interpretation and introduces an entirely new category to the existing hierarchy of quantum correlations. Answering such questions will help us better understand the uniquely quantum incompatibility between spatial and temporal correlations.

Acknowledgments—This work is supported by the National Research Foundation, Singapore, and Agency for Science, Technology and Research (A*STAR) under its QEP2.0 programme (NRF2021-QEP2-02-P06) and its CQT Bridging Grant, the Singapore Ministry of Education Tier 1 Grants No. RG77/22 and No. RT4/23, the Singapore Ministry of Education Tier 2 Grant No. MOE-T2EP50221-0005, Grant No. FQXi-RFP-1809 ("The Role of Quantum Effects in Simplifying Quantum Agents") from the Foundational Questions Institute and Fetzer Franklin Fund (a donor-advised fund of Silicon Valley Community Foundation). T. J. E. is supported by the University of Manchester Dame Kathleen Ollerenshaw Fellowship. V. N. acknowledges support from the Lee Kuan Yew Endowment Fund (Postdoctoral Fellowship).

[1] H.-P. Breuer, E.-M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).
[2] S. Milz and K. Modi, PRX Quantum **2**, 030201 (2021).
[3] O. Oreshkov, F. Costa, and C. Brukner, Nat. Commun. **3**, 1 (2012).
[4] K. Ried, M. Agnew, L. Vermeyden, D. Janzing, R. W. Spekkens, and K. J. Resch, Nat. Phys. **11**, 414 (2015).
[5] J. F. Fitzsimons, J. A. Jones, and V. Vedral, Sci. Rep. **5**, 18281 (2015).
[6] J. M. Kübler and D. Braun, New J. Phys. **20**, 083015 (2018).
[7] M. Hu and Y. Hou, Phys. Rev. A **97**, 062125 (2018).
[8] C. Zhang, Y. Hou, and D. Song, Phys. Rev. A **101**, 062103 (2020).
[9] Z. Zhao, R. Pisarczyk, J. Thompson, M. Gu, V. Vedral, and J. F. Fitzsimons, Phys. Rev. A **98**, 052312 (2018).
[10] F. Costa and S. Shrapnel, New J. Phys. **18**, 063032 (2016).
[11] J.-M. A. Allen, J. Barrett, D. C. Horsman, C. M. Lee, and R. W. Spekkens, Phys. Rev. X **7**, 031021 (2017).
[12] A general quantum process describes evolution of an open system in contact with an external environment. Markovian evolution then aligns with the special case where optimal modeling of the systems future behavior requires only knowledge of its current state ρ_0, whereby its future state ρ_t can be described by quantum channels (completely positive and trace-preserving maps) acting on ρ_0. A non-Markovian process comes about when one can gain more information about ρ_t by knowing the state of the environment—as this signifies that information about the system-environment correlations established at earlier times ($t < 0$) can be fed back into the system. A signature of non-Markovianty is thus when the mapping between ρ_0 and ρ_t cannot be described by a quantum channel.
[13] Throughout this Letter, we denote operators acting on Hilbert spaces of states by boldface letters.

[14] R. Pisarczyk, Z. Zhao, Y. Ouyang, V. Vedral, and J. F. Fitzsimons, Phys. Rev. Lett. **123**, 150502 (2019).

[15] S. Utagi, Phys. Lett. A **386**, 126983 (2021).

[16] Z. Jia, M. Song, and D. Kaszlikowski, New J. Phys. **25**, 123038 (2023).

[17] J. Fullwood, arXiv:2304.03954.

[18] C. Marletto, V. Vedral, S. Virzi, E. Rebufello, A. Avella, F. Piacentini, M. Gramegna, I. P. Degiovanni, and M. Genovese, Nat. Commun. **10**, 182 (2019).

[19] C. Marletto, V. Vedral, S. Virzì, A. Avella, F. Piacentini, M. Gramegna, I. P. Degiovanni, and M. Genovese, Sci. Adv. **7**, eabe4742 (2021).

[20] C. Marletto, V. Vedral, S. Virzì, E. Rebufello, A. Avella, F. Piacentini, M. Gramegna, I. P. Degiovanni, and M. Genovese, Entropy **22**, 228 (2020).

[21] X. Liu, Z. Jia, Y. Qiu, F. Li, and O. Dahlsten, New J. Phys. **26**, 033008 (2024).

[22] X. Liu, Y. Qiu, O. Dahlsten, and V. Vedral, arXiv:2303.10544.

[23] X. Liu, Q. Chen, and O. Dahlsten, Phys. Rev. A **109**, 032219 (2024).

[24] See Supplemental Material, which includes Ref. [25], at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.110202 contains examples, proofs of technical results, and a semidefinite program to compute atemporality.

[25] A. Brodutch, A. Datta, K. Modi, A. Rivas, and C. A. Rodríguez-Rosario, Phys. Rev. A **87**, 042301 (2013).

[26] M.-D. Choi, Linear Algebra Appl. **10**, 285 (1975).

[27] D. Horsman, C. Heunen, M. F. Pusey, J. Barrett, and R. W. Spekkens, Proc. R. Soc. A **473**, 20170395 (2017).

[28] V. Bužek, M. Hillery, and F. Werner, J. Mod. Opt. **47**, 211 (2000).

[29] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).

[30] L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001).

[31] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. **84**, 1655 (2012).

[32] R. F. Werner, Phys. Rev. A **40**, 4277 (1989), the Werner states are a well-studied class of quantum states on bipartite systems AB with subsystems of equal dimensions. A Werner state $\boldsymbol{\rho}_{AB}$ satisfies $\boldsymbol{\rho}_{AB} = (\mathbf{U} \otimes \mathbf{U})\boldsymbol{\rho}_{AB}(\mathbf{U}^\dagger \otimes \mathbf{U}^\dagger)$ for all unitaries $\mathbf{U}$. For two-qubit case, these states can be represented parametrically as $\boldsymbol{\rho}_q^{\text{Werner}} = (q/3)\mathbf{P}_s + (1-q)\mathbf{P}_a$, where $q \in [0, 1]$ and $\mathbf{P}_s$, $\mathbf{P}_a$ are the projectors onto the symmetric and the antisymmetric space, respectively.

[33] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002).

[34] J. Fullwood and A. J. Parzygnat, arXiv:2405.17555.

[35] T. Zhang, O. Dahlsten, and V. Vedral, New J. Phys. **22**, 023029 (2020).

[36] J. M. Dominy and D. A. Lidar, Quantum Inf. Process. **15**, 1349 (2016).

[37] P. Pechukas, Phys. Rev. Lett. **73**, 1060 (1994).

[38] G. A. Paz-Silva, M. J. W. Hall, and H. M. Wiseman, Phys. Rev. A **100**, 042120 (2019).

[39] X.-M. Lu, Phys. Rev. A **93**, 042332 (2016).

[40] K. Modi, C. A. Rodríguez-Rosario, and A. Aspuru-Guzik, Phys. Rev. A **86**, 064102 (2012).

[41] F. Buscemi, Phys. Rev. Lett. **113**, 140502 (2014).

[42] B. Vacchini and G. Amato, Sci. Rep. **6**, 37328 (2016).

[43] A. J. Parzygnat, Electron. Proc. Theor. Comput. Sci. **343**, 1 (2021).

[44] A. J. Parzygnat and B. P. Russo, Linear Algebra Appl. **644**, 28 (2022).

[45] A. J. Parzygnat and J. Fullwood, PRX Quantum **4**, 020334 (2023).

[46] A. J. Parzygnat and F. Buscemi, Quantum **7**, 1013 (2023).

[47] B. Regula, R. Takagi, and M. Gu, Quantum **5**, 522 (2021).

[48] H. A. Carteret, D. R. Terno, and K. Życzkowski, Phys. Rev. A **77**, 042113 (2008).

[49] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007).